# Written Exam for the B.Sc. in Economics autumn 2011-2012 

## Microeconomics C

Final Exam

January 17, 2012
(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

PLEASE ANSWER ALL QUESTIONS.

## PLEASE EXPLAIN YOUR ANSWERS.

1. (a) Solve the game below by iterated elimination of strictly dominated strategies. Describe briefly each step.

|  | $X$ | $Y$ | $Z$ |
| :---: | :---: | :---: | :---: |
| $A$ | 2,2 | 1,4 | 3,1 |
| $B$ | 9,3 | 2,4 | 0,3 |
| $C$ | 3,0 | 0,1 | 0,5 |
| $D$ | 1,7 | 0,1 | 2,2 |

(b) Find all pure and mixed Nash equilibria in the following game:

|  | $t_{1}$ | $t_{2}$ |
| :--- | :--- | :--- |
| $s_{1}$ | 2,1 | 3,0 |
| $s_{2}$ | 1,2 | 4,3 |
| $s_{3}$ | 0,1 | 0,3 |

(c) Consider the game given by the following game tree:

i. Is it a game of perfect or imperfect information? How many subgames are there in the game (excluding the game itself)? What are the possible strategies for the three players?
ii. Find all (pure strategy) subgame perfect Nash equilibria.
iii. Is the strategy profile $\left(R, L^{\prime} R^{\prime}, R^{\prime \prime}\right)$ a Nash equilibrium?
2. Three students $(i=1,2,3)$ are working on a joint project. The amount of time student $i$ spends on the project is denoted $x_{i} \geq 0$. The final quality $q$ of the project depends on $x_{1}, x_{2}$, and $x_{3}$ in the following way:

$$
q\left(x_{1}, x_{2}, x_{3}\right)=2 x_{1}+2 x_{2}+x_{3}-x_{1} x_{2}-x_{1} x_{3} .
$$

Spending time on the project is costly for the students. The cost function for each student is:

$$
c_{i}\left(x_{i}\right)=\left(x_{i}\right)^{2} .
$$

The utility for each student is equal to the final quality of the project minus his cost:

$$
u_{i}\left(x_{1}, x_{2}, x_{3}\right)=q\left(x_{1}, x_{2}, x_{3}\right)-c_{i}\left(x_{i}\right) .
$$

(a) Find the best response functions for the three students. I.e., for each student $i$, find the optimal amount of time to spend on the project given the time spent by the other two students.
(b) Suppose the students simultaneously and independently decide how much time to spend on the project. Find the Nash equilibrium of this game.
(c) Now consider the following two stage game: First, student 1 decides how much time to spend on the project. Finally, after observing the choice of student 1 , the students 2 and 3 simultaneously and independently decide how much time to spend. Set up the maximization problem facing student 1 in stage one.
(d) Find the subgame perfect Nash equilibrium of the two stage game from (c). Who works more and who works less than in the Nash equilibrium from (b)? Give an intuitive explanantion.
3. Consider the following signalling game:

(a) Find a separating perfect Bayesian equilibrium.
(b) Find a pooling perfect Bayesian equilibrium. Does it satisfy Signalling Requirement 5 from Gibbons? Does it satisfy Signalling Requirement 6 ?
4. Consider the bargaining problem $(U, d)$ given by:

$$
\begin{aligned}
U & =\left\{\left(v_{1}, v_{2}\right) \mid v_{1}, v_{2} \geq 0 \text { and } v_{2} \leq-2 v_{1}+4\right\} \\
d & =(0,0)
\end{aligned}
$$

(a) Draw a sketch of $U$. Can the symmetry axiom (SYM) be used to conclude that the Nash bargaining solution of $(U, d)$ must satisfy $v_{1}=v_{2}$ ?
(b) Find the Nash bargaining solution of $(U, d)$.

